```
one_one_corr_tran  THM

⊢  (A ∼ B) ⇒ (B ∼ C) ⇒ (A ∼ C)
| by (Unfold ‘one_one_corr‘ 0) THEN GenExRepD
|
1. A : Type
2. B : Type
3. C : Type
4. f1 : A→B
5. g1 : B→A
6. InvFuns(A; B; f1; g1)
7. f : B→C
8. g : C→B
9. InvFuns(B; C; f; g)
⊢  ∃f:(A→C), g:(C→A). InvFuns(A; C; f; g)
| by InstConcl [f o f1;g1 o g]
|
⊢  InvFuns(A; C; f o f1; g1 o g)
| by (Unfold ‘inv_funs‘ 0) THEN (Unfold ‘inv_funs‘ 9)
|    THEN
|    (Unfold ‘inv_funs‘ 6)
|    THEN
|    ExRepD
|
6. g1 o f1 = Id
7. f1 o g1 = Id
8. f : B→C
9. g : C→B
10. g o f = Id
11. f o g = Id
⊢  g1 o g o f o f1 = Id & f o f1 o g1 o g = Id
| by (Analyze 0) THEN Ext THEN (Reduce 0)
|
|\
| 12. x : A
| ⊢  g1(g(f(f1(x)))) = x  by RWNth 2 add_composeC 0
| |
| ⊢  g1((g o f)(f1(x))) = x  by (HypSubst 10 0) THEN (Reduce 0)
| |
| ⊢  g1(f1(x)) = x  by RWH add_composeC 0
| |
| ⊢  (g1 o f1)(x) = x  by (HypSubst 6 0) THEN (Reduce 0)
|
.\
. 12. x : C
. ⊢  f(f1(g1(g(x)))) = x  by RWNth 2 add_composeC 0
. |
. ⊢  f((f1 o g1)(g(x))) = x  by (HypSubst 7 0) THEN (Reduce 0)
. |
. ⊢  f(g(x)) = x  by RWH add_composeC 0
. |
. ⊢  (f o g)(x) = x  by (HypSubst 11 0) THEN (Reduce 0)
END
```