```
preima_of_equiv_rel  THM

⊢  ∀f:(A→B), R:(B→B→Prop).
|  (EquivRel x,y:B. x R y) ⇒ (EquivRel x,y:A. x R_f y)
| by (Unfolds [‘equiv_rel‘;‘preima_of_rel‘] 0)
|    THEN
|    (Unfolds [‘refl‘;‘sym‘;‘trans‘] 0)
|
⊢  ∀A,B:Type, f:(A→B), R:(B→B→Prop).
|  (∀a:B. a R a)
|  & (∀a,b:B. (a R b) ⇒ (b R a))
|  & (∀a,b,c:B. (a R b) ⇒ (b R c) ⇒ (a R c))
|  ⇒
|
|  (∀a:A. a (λx,y. (f(x)) R (f(y))) a)
|  & (∀a,b:A.
|     (a (λx,y. (f(x)) R (f(y))) b) ⇒ (b (λx,y. (f(x)) R (f(y))) a))
|  & (∀a,b,c:A.
|     (a (λx,y. (f(x)) R (f(y))) b) ⇒
|     (b (λx,y. (f(x)) R (f(y))) c) ⇒ (a (λx,y. (f(x)) R (f(y))) c))
| by (Reduce 0) THEN GenExRepD
|
|\
| 1. A : Type
| 2. B : Type
| 3. f : A→B
| 4. R : B→B→Prop
| 5. ∀a:B. a R a
| 6. ∀a,b:B. (a R b) ⇒ (b R a)
| 7. ∀a,b,c:B. (a R b) ⇒ (b R c) ⇒ (a R c)
| 8. a : A
| ⊢  (f(a)) R (f(a))  by BackThru 5
|
|\
| 1. A : Type
| 2. B : Type
| 3. f : A→B
| 4. R : B→B→Prop
| 5. ∀a:B. a R a
| 6. ∀a,b:B. (a R b) ⇒ (b R a)
| 7. ∀a,b,c:B. (a R b) ⇒ (b R c) ⇒ (a R c)
| 8. a : A
| 9. b : A
| 10. (f(a)) R (f(b))
| ⊢  (f(b)) R (f(a))  by BackThru 6
|
.\
. 1. A : Type
. 2. B : Type
. 3. f : A→B
. 4. R : B→B→Prop
. 5. ∀a:B. a R a
. 6. ∀a,b:B. (a R b) ⇒ (b R a)
. 7. ∀a,b,c:B. (a R b) ⇒ (b R c) ⇒ (a R c)
. 8. a : A
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. 9. b : A
. 10. c : A
. 11. (f(a)) R (f(b))
. 12. (f(b)) R (f(c))
. ⊢  (f(a)) R (f(c))  by FwdThru 7 [11;12]
END
```