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quo_of_quo  THM

⊢  ∀R:(T→T→Prop).
|  (EquivRel x,y:T. x R y) ⇒
|  (∀Q:((x,y:T//(x R y))→(x,y:T//(x R y))→Prop).
|   (EquivRel u,v:x,y:T//(x R y). u Q v) ⇒
|   ((x,y:T//(x Q y)) ∼ (u,v:(x,y:T//(x R y))//(u Q v))))
| by UnivCD THEN (Unfold `one_one_corr` 0)
|
1. T : Type
2. R : T→T→Prop
3. EquivRel x,y:T. x R y
4. Q : (x,y:T//(x R y))→(x,y:T//(x R y))→Prop
5. EquivRel u,v:x,y:T//(x R y). u Q v
⊢  ∃f:((x,y:T//(x Q y))→(u,v:(x,y:T//(x R y))//(u Q v)))
|  , g:((u,v:(x,y:T//(x R y))//(u Q v))→(x,y:T//(x Q y))).
|  InvFuns(x,y:T//(x Q y); u,v:(x,y:T//(x R y))//(u Q v); f; g)
| by Assert (EquivRel x,y:T. x Q y)
|
|\
| .....assertion .....
| ⊢  EquivRel x,y:T. x Q y
|   by BackThru
|      Thm* ∀R:(T→T→Prop).
|           (EquivRel x,y:T. x R y) ⇒
|           (∀Q:((x,y:T//(x R y))→(x,y:T//(x R y))→Prop).
|            (EquivRel u,v:x,y:T//(x R y). u Q v) ⇒
|            (EquivRel x,y:T. x Q y))
|      {incl_aux6_quo}
|
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. 6. EquivRel x,y:T. x Q y
. ⊢   by Let (f = (λx.x))
. |
. |\
. | 7. x : x,y:T//(x Q y)
. | ⊢  x ∈ u,v:(x,y:T//(x R y))//(u Q v)
. |   by (Unfold `member` 0) THEN (Analyze 7) THEN Analyze
. |
. .\
. . 7. f : (x,y:T//(x Q y))→(u,v:(x,y:T//(x R y))//(u Q v))
. . 8. f = (λx.x)
. . ⊢   by Let (g = (λx.x))
. . |
. . |\
. . | 9. x : u,v:(x,y:T//(x R y))//(u Q v)
. . | ⊢  x ∈ x,y:T//(x Q y)  by (Unfold `member` 0) THEN (Analyze 9)
. . | |
. . | 9. x1 : x,y:T//(x R y)
. . | 10. x2 : x,y:T//(x R y)
. . | 11. x1 Q x2
. . | ⊢  x1 = x2  by QuotD 9
. . | |
. . | 9. x3 : T
. . | 10. x4 : T
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. . | 11. x3 R x4
. . | 12. x2 : x,y:T//(x R y)
. . | 13. x3 Q x2
. . | ⊢  x3 = x2  by QuotD 12
. . | |
. . | 12. x5 : T
. . | 13. x6 : T
. . | 14. x5 R x6
. . | 15. x3 Q x5
. . | ⊢  x3 = x6  by Analyze
. . | |
. . | ⊢  x3 Q x6  by Assert (x5 = x6)
. . | |
. . | |\
. . | | .....assertion .....
. . | | ⊢  x5 = x6  by Analyze
. . | |
. . | .\
. . | . 16. x5 = x6
. . | . ⊢   by HypSubst 16 15
. . |
. . .\
. . . 9. g : (u,v:(x,y:T//(x R y))//(u Q v))→(x,y:T//(x Q y))
. . . 10. g = (λx.x)
. . . ⊢   by InstConcl [f;g]
. . . |
. . . ⊢  InvFuns(x,y:T//(x Q y); u,v:(x,y:T//(x R y))//(u Q v); f; g)
. . . | by Unfold `inv_funs` 0
. . . |
. . . ⊢  g o f = Id & f o g = Id
. . . | by (Analyze 0) THEN (RWW "8 10" 0) THEN Ext THEN (Reduce 0)
. . . |
. . . |\
. . . | 11. x : x,y:T//(x Q y)
. . . | ⊢  x = x  by Auto
. . . |
. . . .\
. . . . 11. x : u,v:(x,y:T//(x R y))//(u Q v)
. . . . ⊢  x = x  by Auto
END
```