```
quotient_1_1_corr  THM

⊢  ∀f:(A→B), R:(B→B→Prop).
|  Bij(A; B; f) ⇒
|  (EquivRel x,y:B. x R y) ⇒
|  (∃F:((x,y:A//(x R_f y))→(x,y:B//(x R y))).
|   Bij(x,y:A//(x R_f y); x,y:B//(x R y); F))  by UnivCD
|
1. A : Type
2. B : Type
3. f : A→B
4. R : B→B→Prop
5. Bij(A; B; f)
6. EquivRel x,y:B. x R y
⊢  ∃F:((x,y:A//(x R_f y))→(x,y:B//(x R y))).
|  Bij(x,y:A//(x R_f y); x,y:B//(x R y); F)
| by Inst
|    Thm* ∀f:(A→B), R:(B→B→Prop).
|         (EquivRel x,y:B. x R y) ⇒ (EquivRel x,y:A. x R_f y)
|    {preima_of_equiv_rel}
|    [A;B;f;R]
|
7. EquivRel x,y:A. x R_f y
⊢
| by (FwdThru
|     Thm* ∀f:(A→B). Bij(A; B; f) ⇒ (∃g:(B→A). InvFuns(A; B; f; g))
|     {bij_imp_exists_inv}
|     [5])
|    THEN
|    (Analyze -1)
|
8. g : B→A
9. InvFuns(A; B; f; g)
⊢   by (Let (F = (λx.f(x)))) THEN (Let (G = g))
|
|\
| .....wf .....
| ⊢  (λx.f(x)) ∈ (x,y:A//(x R_f y))→(x,y:B//(x R y))
| | by EqToMemberEq (λi.EqCDAux ‘strong‘) 0
| |
| |\
| | 10. x : x,y:A//(x R_f y)
| | ⊢  f(x) ∈ x,y:B//(x R y)  by Analyze -1
| | |
| | 10. x1 : A
| | 11. x2 : A
| | 12. x1 R_f x2
| | ⊢  f(x1) = f(x2)  by Unfold ‘preima_of_rel‘ -1 THEN Reduce -1
| | |
| | 12. (f(x1)) R (f(x2))
| | ⊢   by Analyze
| |
| .\
| . .....eq aux .....
| . ⊢  x,y:A//(x R_f y) ∈ Type  by Auto
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|
|\
| .....wf .....
| ⊢  (x,y:A//(x R_f y))→(x,y:B//(x R y)) ∈ Type  by Auto
|
|\
| .....wf .....
| 10. F : (x,y:A//(x R_f y))→(x,y:B//(x R y))
| 11. F = (λx.f(x))
| ⊢  g ∈ (x,y:B//(x R y))→(x,y:A//(x R_f y))  by ExtWith [] [B→A]
| |
| 12. x : x,y:B//(x R y)
| ⊢  g(x) ∈ x,y:A//(x R_f y)
| | by (Unfold `member` 0) THEN (Analyze 12) THEN Analyze
| |
| 12. x1 : B
| 13. x2 : B
| 14. x1 R x2
| ⊢  (g(x1)) R_f (g(x2))
| | by (Unfold `preima_of_rel` 0) THEN (Reduce 0)
| |    THEN
| |    (RWH add_composeC 0)
| |
| ⊢  ((f o g)(x1)) R ((f o g)(x2))
|   by (Analyze 9) THEN (HypSubst 10 0) THEN (Reduce 0)
|
|\
| .....wf .....
| 10. F : (x,y:A//(x R_f y))→(x,y:B//(x R y))
| 11. F = (λx.f(x))
| ⊢  (x,y:B//(x R y))→(x,y:A//(x R_f y)) ∈ Type  by Auto
|
.\
. 10. F : (x,y:A//(x R_f y))→(x,y:B//(x R y))
. 11. F = (λx.f(x))
. 12. G : (x,y:B//(x R y))→(x,y:A//(x R_f y))
. 13. G = g
. ⊢
. | by Inst
. |    Thm* ∀f:(A→B), g:(B→A). InvFuns(A; B; f; g) ⇒ Bij(A; B; f)
. |    {fun_with_inv_is_bij}
. |    [x,y:A//(x R_f y);x,y:B//(x R y);F;G]
. |
. |\
. | .....antecedent .....
. | ⊢  InvFuns(x,y:A//(x R_f y); x,y:B//(x R y); F; G)
. | | by (Analyze 0) THEN Ext THEN (RWW "11 13" 0) THEN (Reduce 0)
. | |
. | |\
. | | 14. x : x,y:A//(x R_f y)
. | | ⊢  g(f(x)) = x  by (Analyze 14) THEN Analyze
. | | |
. | | 14. x1 : A
. | | 15. x2 : A
. | | 16. x1 R_f x2
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. | | ⊢  (g(f(x1))) R_f x2  by RWH add_composeC 0
. | | |
. | | ⊢  ((g o f)(x1)) R_f x2
. | |   by (Analyze 9) THEN (HypSubst 9 0) THEN (Reduce 0)
. | |
. | .\
. | . 14. x : x,y:B//(x R y)
. | . ⊢  f(g(x)) = x  by (Analyze 14) THEN Analyze
. | . |
. | . 14. x1 : B
. | . 15. x2 : B
. | . 16. x1 R x2
. | . ⊢  (f(g(x1))) R x2
. | .   by (RWH add_composeC 0) THEN (Analyze 9) THEN (HypSubst 10 0)
. | .      THEN
. | .      (Reduce 0)
. |
. .\
. . 14. Bij(x,y:A//(x R_f y); x,y:B//(x R y); F)
. . ⊢   by InstConcl [F]
END
```