```
quotient_of_finite  THM

⊢  ∀n:{1...}, A:Type, R:(A→A→Prop).
|  (ℕn ∼ A) ⇒
|  (EquivRel x,y:A. x R y) ⇒
|  (∀x,y:A. Dec(x R y)) ⇒ (∃m:ℕ(n+1). ℕm ∼ (x,y:A//(x R y)))
| by UnivCD
|
1. n : {1...}
2. A : Type
3. R : A→A→Prop
4. ℕn ∼ A
5. EquivRel x,y:A. x R y
6. ∀x,y:A. Dec(x R y)
⊢  ∃m:ℕ(n+1). ℕm ∼ (x,y:A//(x R y))
| by (FwdThru
|     Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)    {bij_iff_1_1_corr}
|     [4])
|    THEN
|    (Analyze -1)
|
7. f : ℕn→A
8. Bij(ℕn; A; f)
⊢
| by FwdThru
|    Thm* ∀f:(A→B), R:(B→B→Prop).
|         Bij(A; B; f) ⇒
|         (EquivRel x,y:B. x R y) ⇒
|         (∃F:((x,y:A//(x R_f y))→(x,y:B//(x R y))).
|          Bij(x,y:A//(x R_f y); x,y:B//(x R y); F))
|    {quotient_1_1_corr}
|    [5;8]
|
9. ∃F:((x,y:ℕn//(x R_f y))→(x,y:A//(x R y))).
|  Bij(x,y:ℕn//(x R_f y); x,y:A//(x R y); F)
⊢
| by Inst
|    Thm* ∀f:(A→B), R:(B→B→Prop).
|         (EquivRel x,y:B. x R y) ⇒ (EquivRel x,y:A. x R_f y)
|    {preima_of_equiv_rel}
|    [ℕn;A;f;R]
|
10. EquivRel x,y:ℕn. x R_f y
⊢
| by FwdThru
|    Thm* (∃f:(A→B). Bij(A; B; f)) ⟺ (A ∼ B)    {bij_iff_1_1_corr}
|    [9]
|
11. (x,y:ℕn//(x R_f y)) ∼ (x,y:A//(x R y))
⊢
| by Inst
|    Thm* ∀n:{1...}, E:(ℕn→ℕn→Prop).
|         (EquivRel x,y:ℕn. x E y) & (∀x,y:ℕn. Dec(x E y)) ⇒
|         (∃m:ℕ(n+1). ℕm ∼ (i,j:ℕn//(i E j)))
|    {quotient_of_nsubn}
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|    [n;R_f]
|
|\
| .....decidable? .....
| 12. x : ℕn
| 13. y : ℕn
| ⊢  Dec(x R_f y)  by (Unfold ‘preima_of_rel‘ 0) THEN (Reduce 0)
|
.\
. 12. ∃m:ℕ(n+1). ℕm ∼ (i,j:ℕn//(i R_f j))
. ⊢   by (Analyze 12) THEN (InstConcl [m])
. |
. 12. m : ℕ(n+1)
. 13. ℕm ∼ (i,j:ℕn//(i R_f j))
. ⊢  ℕm ∼ (x,y:A//(x R y))
.   by FwdThru
.      Thm* (A ∼ B) ⇒ (B ∼ C) ⇒ (A ∼ C)    {one_one_corr_tran}
.      [13;11]
END
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