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fincr_formation  THM

⊢  FIncr  by Unfold ‘fincr‘ 0
|
⊢  {f | i:ℕ → if i=₂0→ ℤ else {f(i-1)...} fi}  by Witness λi,j. i<j
|
|\
| .....aux .....
| ⊢  ↓WellFnd{1}(ℕ;u,v.(λi,j. i<j)(u,v))
| | by RW
| |    (RepeatC (UnfoldsC [‘wellfounded‘;‘guard‘;‘so_apply‘])
| |     THENC
| |     AbReduceC)
| |    0
| |    THEN
| |    Analyze 0
| |
| 1. P : ℕ→Prop
| 2. ∀j:ℕ. (∀k:ℕ. k<j ⇒ P(k)) ⇒ P(j)
| 3. n : ℕ
| ⊢  P(n)  by CompNatInd -1
| |
| 4. ∀n1:ℕ. n1<n ⇒ P(n1)
| ⊢  P(n)  by ((With n (Analyze -3)) THEN (Analyze -1)) THEN Trivial
|
.\
. 1. i : ℕ
. 2. f :
. |  {f | i:{i1:ℕ| i1 (λi,j. i<j) i }
. |       → if i=₂0→ ℤ else {f(i-1)...} fi}
. ⊢  if i=₂0→ ℤ else {f(i-1)...} fi
. | by Assert (∀j:{k:ℕ| k<i }. f(j) ∈ ℤ)
. |
. |\
. | .....assertion .....
. | ⊢  ∀j:{k:ℕ| k<i }. f(j) ∈ ℤ
. | | by (AbReduce -1 THEN Analyze 0) THEN (Analyze -1)
. | |    THEN
. | |    (CompNatInd -2)
. | |
. | 2. f : {f | i:{i1:ℕ| i1<i } → if i=₂0→ ℤ else {f(i-1)...} fi}
. | 3. j : ℕ
. | 4. ∀j1:ℕ. j1<j ⇒ j1<i ⇒ f(j1) ∈ ℤ
. | ⊢  j<i ⇒ f(j) ∈ ℤ  by Analyze 0
. | |
. | 5. j<i
. | ⊢  f(j) ∈ ℤ  by Witness2 j
. | |
. | 6. y : if j=₂0→ ℤ else {f(j-1)...} fi
. | 7. y = f(j)
. | ⊢  f(j) ∈ ℤ  by SplitITE -1 THEN Try (Auto THEN Fail)
. | |
. | .....falsecase .....
. | 7. y = f(j)
. | 8. j ≠ 0
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. | ⊢  f(j) ∈ ℤ
. | | by (Witness4 j-1 THEN (Analyze -1) THEN (Analyze -1))
. | |    THENA
. | |    (Thin 6 THEN Thin 5 THEN Thin 2)
. | |
. | 4. j<i
. | 5. y : if j=₂0→ ℤ else {f(j-1)...} fi
. | 6. y = f(j)
. | 7. j ≠ 0
. | 8. f(j-1) ∈ ℤ
. | ⊢  f(j) ∈ ℤ
. | | by Assert (∀i:ℤ, j:{i...}. j ∈ ℤ) THENA OnAllHyps Thin
. | |
. | 9. ∀i:ℤ, j:{i...}. j ∈ ℤ
. | ⊢  f(j) ∈ ℤ
. |   by (With (f(j-1)) (Analyze -1)) THEN (With (f(j)) (Analyze -1))
. |
. .\
. . 3. ∀j:{k:ℕ| k<i }. f(j) ∈ ℤ
. . ⊢  if i=₂0→ ℤ else {f(i-1)...} fi  by SplitITE 0
. . |
. . |\
. . | .....truecase .....
. . | 4. i = 0
. . | ⊢  ℤ  by Witness 1
. . |
. . .\
. . . .....falsecase .....
. . . 4. i ≠ 0
. . . ⊢  {f(i-1)...}  by Witness f(i-1)+1
. . . |
. . . |\
. . . | .....wf .....
. . . | ⊢  f(i-1)+1 ∈ ℤ  by Analyze
. . . | |
. . . | |\
. . . | | ⊢  f(i-1) ∈ ℤ  by With (i-1) (Analyze -2)
. . . | |
. . . | .\
. . . | . ⊢  1 ∈ ℤ  by Auto
. . . |
. . . |\
. . . | ⊢  f(i-1)≤f(i-1)+1  by Auto
. . . | |
. . . | |\
. . . | | ⊢  f(i-1) ∈ ℤ  by With (i-1) (Analyze -2)
. . . | |

. . . | .\
. . . | . .....wf .....
. . . | . ⊢  f(i-1) ∈ ℤ  by With (i-1) (Analyze -2)
. . . |
. . . .\
. . . . .....wf .....
. . . . 5. j : ℤ
. . . . ⊢  (f(i-1)≤j) ∈ Prop  by Analyze
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. . . . |\
. . . . | ⊢  f(i-1) ∈ ℤ  by With (i-1) (Analyze -3)
. . . . |
. . . . .\
. . . . . ⊢  j ∈ ℤ  by Trivial
END
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