```
fincr_wf  THM

⊢  FIncr ∈ Type  by Unfold ‘fincr‘ 0
|
⊢  {f | i:ℕ → if i=₂0→ ℤ else {f(i-1)...} fi} ∈ Type
| by With (λi,j. i<j) Analyze
|
|\
| ⊢  ℕ ∈ Type  by Auto
|
|\
| ⊢  (λi,j. i<j) ∈ ℕ→ℕ→Type  by Auto
|
|\
| .....eq aux .....
| ⊢  ↓WellFnd{[1]}(ℕ;u,v.(λi,j. i<j)(u,v))
| | by RW
| |    (RepeatC (UnfoldsC [‘wellfounded‘;‘guard‘;‘so_apply‘])
| |     THENC
| |     AbReduceC)
| |    0
| |    THEN
| |    Analyze 0
| |    THEN
| |    UnivCD
| |
| 1. P : ℕ→Prop
| 2. ∀j:ℕ. (∀k:ℕ. k<j ⇒ P(k)) ⇒ P(j)
| 3. n : ℕ
| ⊢  P(n)  by CompNatInd -1
| |
| 4. ∀n1:ℕ. n1<n ⇒ P(n1)
| ⊢  P(n)  by Witness2 n
| |
| 2. n : ℕ
| 3. ∀n1:ℕ. n1<n ⇒ P(n1)
| 4. (∀k:ℕ. k<n ⇒ P(k)) ⇒ P(n)
| ⊢  P(n)  by Analyze -1 THEN Trivial
|
.\
. .....eq aux .....
. 1. i : ℕ
. 2. f :
. |  {f | i:{z:ℕ| z (λi,j. i<j) i } → if i=₂0→ ℤ else {f(i-1)...} fi}
. ⊢  if i=₂0→ ℤ else {f(i-1)...} fi ∈ Type
. | by AbReduce -1 THEN Assert (∀j:{k:ℕ| k<i }. f(j) ∈ ℤ)
. |
. |\
. | .....assertion .....
. | 2. f : {f | i:{z:ℕ| z<i } → if i=₂0→ ℤ else {f(i-1)...} fi}
. | ⊢  ∀j:{k:ℕ| k<i }. f(j) ∈ ℤ  by Analyze 0
. | |
. | 3. j : {k:ℕ| k<i }
. | ⊢  f(j) ∈ ℤ  by Analyze -1 THEN CompNatInd -2
. | |
```

```
. | 3. j : ℕ
. | 4. ∀j1:ℕ. j1<j ⇒ j1<i ⇒ f(j1) ∈ ℤ
. | ⊢  j<i ⇒ f(j) ∈ ℤ  by Analyze 0
. | |
. | 5. j<i
. | ⊢  f(j) ∈ ℤ  by Witness2 j
. | |
. | 6. y : if j=₂0→ ℤ else {f(j-1)...} fi
. | 7. y = f(j)
. | ⊢  f(j) ∈ ℤ  by SplitITE -1 THEN Try (Auto THEN Fail)
. | |
. | .....falsecase .....
. | 7. y = f(j)
. | 8. j ≠ 0
. | ⊢  f(j) ∈ ℤ
. | | by Assert (∀i:ℤ, j:{i...}. j ∈ ℤ) THENA OnAllHyps Thin
. | |
. | 9. ∀i:ℤ, j:{i...}. j ∈ ℤ
. | ⊢  f(j) ∈ ℤ  by With (f(j-1)) (Analyze -1)
. | |
. | |\
. | | .....wf .....
. | | 8. j ≠ 0
. | | ⊢  f(j-1) ∈ ℤ
. | |   by (With (j-1) (Analyze -5)) THEN (Analyze -1)
. | |      THEN
. | |      (Analyze -1)
. | |
. | .\
. | . 9. ∀j:{f(j-1)...}. j ∈ ℤ
. | . ⊢  f(j) ∈ ℤ  by With (f(j)) (Analyze -1)
. |
. .\
. . 2. f : {f | i:{z:ℕ| z<i } → if i=₂0→ ℤ else {f(i-1)...} fi}
. . 3. ∀j:{k:ℕ| k<i }. f(j) ∈ ℤ
. . ⊢  if i=₂0→ ℤ else {f(i-1)...} fi ∈ Type
. . | by SplitITE 0 THEN Try (Auto THEN Fail)
. . |
. . .....falsecase .....
. . 4. i ≠ 0
. . ⊢  {f(i-1)...} ∈ Type  by Analyze
. . |
. . ⊢  f(i-1) ∈ ℤ  by With (i-1) (Analyze -2)
END
```