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assignment_monotone  THM

⊢  ∀a:Assignment, f:Formula, a':Assignment.
|  a' extends a ⇒ (a |= f  ⇒ a' |= f ) & (a |≠ f ⇒ a' |≠ f)
| by UnivCD
|
1. a : Assignment
2. f : Formula
3. a' : Assignment
4. a' extends a
⊢  (a |= f  ⇒ a' |= f ) & (a |≠ f ⇒ a' |≠ f)
| by OnHyps [-1;-1;-2] MoveToConcl
|
1. f : Formula
⊢  ∀a,a':Assignment.
|  a' extends a ⇒ (a |= f  ⇒ a' |= f ) & (a |≠ f ⇒ a' |≠ f)
| by FormulaRankInd -1
|
|\
| 2. x : Var
| ⊢  ∀a,a':Assignment.
| |  a' extends a ⇒
| |  (a |= ⌈x⌉  ⇒ a' |= ⌈x⌉ ) & (a |≠ ⌈x⌉ ⇒ a' |≠ ⌈x⌉)
| | by UnivCD THEN Analyze 0 THEN Analyze 0
| |
| |\
| | 3. a : Assignment
| | 4. a' : Assignment
| | 5. a' extends a
| | 6. a |= ⌈x⌉
| | ⊢  a' |= ⌈x⌉
| | | by Inst
| | |    Thm* ∀a,a':Assignment.
| | |         a' extends a ⇒ (∀x:Var. ¬a(x) = 3₁ ⇒ a(x) = a'(x))
| | |    {extension_lemma}
| | |    [a;a';x]
| | |
| | |\
| | | .....antecedent .....
| | | ⊢  ¬a(x) = 3₁  by UnfoldTopAb -1 THEN Reduce -1
| | | |
| | | 6. a(x) = 3₂
| | | ⊢   by Analyze 0
| | | |
| | | 7. a(x) = 3₁
| | | ⊢  False  by HypSubst -1 -2
| | | |
| | | 6. 3₁ = 3₂
| | | 7. a(x) = 3₁
| | | ⊢   by ThreeNEQ -2
| | |
| | .\
| | . 7. a(x) = a'(x)
| | . ⊢   by OnHyps [-2;0] (λi.UnfoldTopAb i THEN Reduce i)
| | . |
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| | . 6. a(x) = 3₂
| | . 7. a(x) = a'(x)
| | . ⊢  a'(x) = 3₂  by HypSubst -1 -2
| |
| .\
| . 3. a : Assignment
| . 4. a' : Assignment
| . 5. a' extends a
| . 6. a |≠ ⌈x⌉
| . ⊢  a' |≠ ⌈x⌉
| . | by Inst
| . |    Thm* ∀a,a':Assignment.
| . |         a' extends a ⇒ (∀x:Var. ¬a(x) = 3₁ ⇒ a(x) = a'(x))
| . |    {extension_lemma}
| . |    [a;a';x]
| . |
| . |\
| . | .....antecedent .....
| . | ⊢  ¬a(x) = 3₁  by UnfoldTopAb -1 THEN Reduce -1
| . | |
| . | 6. a(x) = 3₀
| . | ⊢   by Analyze 0
| . | |
| . | 7. a(x) = 3₁
| . | ⊢  False  by HypSubst -1 -2
| . | |
| . | 6. 3₁ = 3₀
| . | 7. a(x) = 3₁
| . | ⊢   by ThreeNEQ -2
| . |
| . .\
| . . 7. a(x) = a'(x)
| . . ⊢   by OnHyps [-2;0] (λi.UnfoldTopAb i THEN Reduce i)
| . . |
| . . 6. a(x) = 3₀
| . . 7. a(x) = a'(x)
| . . ⊢  a'(x) = 3₀  by HypSubst -1 -2
|
|\
| 2. x : Formula
| 3. ∀a,a':Assignment.
| |  a' extends a ⇒ (a |= x  ⇒ a' |= x ) & (a |≠ x ⇒ a' |≠ x)
| ⊢  ∀a,a':Assignment.
| |  a' extends a ⇒
| |  (a |= ⌈◇⌉x  ⇒ a' |= ⌈◇⌉x ) & (a |≠ ⌈◇⌉x ⇒ a' |≠ ⌈◇⌉x)
| | by UnivCD
| |

| 4. a : Assignment
| 5. a' : Assignment
| 6. a' extends a
| ⊢  (a |= ⌈◇⌉x  ⇒ a' |= ⌈◇⌉x ) & (a |≠ ⌈◇⌉x ⇒ a' |≠ ⌈◇⌉x)
| | by With a (Analyze -4) THEN With a' (Analyze -1) THEN Analyze -1
| |    THEN
| |    Analyze -1
| |
```

```
| 3. a : Assignment
| 4. a' : Assignment
| 5. a' extends a
| 6. a |= x  ⇒ a' |= x
| 7. a |≠ x ⇒ a' |≠ x
| ⊢   by Analyze 0 THEN Analyze 0
| |
| |\
| | 8. a |= ⌈◇⌉x
| | ⊢  a' |= ⌈◇⌉x
| | | by BackThru
| | |    Thm* ∀F:Formula, a:Assignment. a |= ⌈◇⌉F  ⟺ a |≠ F
| | |    {formula_not_sat_lemma}
| | |
| | ⊢  a' |≠ x  by HypBackchain
| | |
| | ⊢  a |≠ x
| |   by FwdThru
| |      Thm* ∀F:Formula, a:Assignment. a |= ⌈◇⌉F  ⟺ a |≠ F
| |      {formula_not_sat_lemma}
| |      [-1]
| |
| .\
| . 8. a |≠ ⌈◇⌉x
| . ⊢  a' |≠ ⌈◇⌉x
| . | by BackThru
| . |    Thm* ∀F:Formula, a:Assignment. a |≠ ⌈◇⌉F ⟺ a |= F
| . |    {formula_not_falsifiable_lemma}
| . |
| . ⊢  a' |= x   by HypBackchain
| . |
| . ⊢  a |= x
| .   by FwdThru
| .      Thm* ∀F:Formula, a:Assignment. a |≠ ⌈◇⌉F ⟺ a |= F
| .      {formula_not_falsifiable_lemma}
| .      [-1]
|
|\
| 2. x1 : Formula
| 3. x2 : Formula
| 4. ∀a,a':Assignment.
| |  a' extends a ⇒ (a |= x1  ⇒ a' |= x1 ) & (a |≠ x1 ⇒ a' |≠ x1)
| 5. ∀a,a':Assignment.
| |  a' extends a ⇒ (a |= x2  ⇒ a' |= x2 ) & (a |≠ x2 ⇒ a' |≠ x2)
| ⊢  ∀a,a':Assignment.
| |  a' extends a ⇒
| |  (a |= x1⌈∧⌉x2  ⇒ a' |= x1⌈∧⌉x2 )
| |  & (a |≠ x1⌈∧⌉x2 ⇒ a' |≠ x1⌈∧⌉x2)  by UnivCD
| |
| 6. a : Assignment
| 7. a' : Assignment
| 8. a' extends a
| ⊢  (a |= x1⌈∧⌉x2  ⇒ a' |= x1⌈∧⌉x2 )
| |  & (a |≠ x1⌈∧⌉x2 ⇒ a' |≠ x1⌈∧⌉x2)
| | by OnHyps [-5;-6]
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| |    (λi.
| |     With a (Analyze i) THEN With a' (Analyze -1) THEN Analyze -1
| |     THEN
| |     Analyze -1)
| |
| 4. a : Assignment
| 5. a' : Assignment
| 6. a' extends a
| 7. a |= x1  ⇒ a' |= x1
| 8. a |≠ x1 ⇒ a' |≠ x1
| 9. a |= x2  ⇒ a' |= x2
| 10. a |≠ x2 ⇒ a' |≠ x2
| ⊢   by Analyze 0 THEN Analyze 0
| |
| |\
| | 11. a |= x1⌈∧⌉x2
| | ⊢  a' |= x1⌈∧⌉x2
| | | by FwdThru
| | |    Thm* ∀a:Assignment, q,r:Formula.
| | |         a |= q⌈∧⌉r  ⟺ a |= q  & a |= r
| | |    {formula_and_sat_lemma}
| | |    [-1]
| | |    THEN
| | |    Sel 2
| | |    (FwdThru
| | |     Thm* ∀a:Assignment, q,r:Formula.
| | |          a |= q⌈∧⌉r  ⟺ a |= q  & a |= r
| | |     {formula_and_sat_lemma}
| | |     [-2])
| | |
| | 12. a |= x1
| | 13. a |= x2
| | ⊢
| |   by BackThru
| |      Thm* ∀a:Assignment, q,r:Formula.
| |           a |= q⌈∧⌉r  ⟺ a |= q  & a |= r
| |      {formula_and_sat_lemma}

| |      THEN
| |      Analyze 0
| |      THEN
| |      HypBackchain
| |
| .\
| . 11. a |≠ x1⌈∧⌉x2
| . ⊢  a' |≠ x1⌈∧⌉x2
| . | by FwdThru
| . |    Thm* ∀a:Assignment, q,r:Formula.
| . |         a |≠ q⌈∧⌉r ⟺ a |≠ q ∨ a |≠ r
| . |    {formula_and_falsifiable_lemma}
| . |    [-1]
| . |    THEN
| . |    Analyze -1
| . |
| . |\
| . | 12. a |≠ x1
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| . | ⊢
| . | | by BackThru
| . | |    Thm* ∀a:Assignment, q,r:Formula.
| . | |         a |≠ q⌈∧⌉r ⟺ a |≠ q ∨ a |≠ r
| . | |    {formula_and_falsifiable_lemma}
| . | |
| . | ⊢  a' |≠ x1 ∨ a' |≠ x2  by Choose [1] THEN HypBackchain
| . |
| . .\
| . . 12. a |≠ x2
| . . ⊢
| . . | by BackThru
| . . |    Thm* ∀a:Assignment, q,r:Formula.
| . . |         a |≠ q⌈∧⌉r ⟺ a |≠ q ∨ a |≠ r
| . . |    {formula_and_falsifiable_lemma}
| . . |
| . . ⊢  a' |≠ x1 ∨ a' |≠ x2  by Choose [2] THEN HypBackchain
|
|\
| 2. x1 : Formula
| 3. x2 : Formula
| 4. ∀a,a':Assignment.
| |  a' extends a ⇒ (a |= x1  ⇒ a' |= x1 ) & (a |≠ x1 ⇒ a' |≠ x1)
| 5. ∀a,a':Assignment.
| |  a' extends a ⇒ (a |= x2  ⇒ a' |= x2 ) & (a |≠ x2 ⇒ a' |≠ x2)
| ⊢  ∀a,a':Assignment.
| |  a' extends a ⇒
| |  (a |= x1⌈∨⌉x2  ⇒ a' |= x1⌈∨⌉x2 )
| |  & (a |≠ x1⌈∨⌉x2 ⇒ a' |≠ x1⌈∨⌉x2)  by UnivCD
| |
| 6. a : Assignment
| 7. a' : Assignment
| 8. a' extends a
| ⊢  (a |= x1⌈∨⌉x2  ⇒ a' |= x1⌈∨⌉x2 )
| |  & (a |≠ x1⌈∨⌉x2 ⇒ a' |≠ x1⌈∨⌉x2)
| | by OnHyps [-5;-6]
| |    (λi.
| |     With a (Analyze i) THEN With a' (Analyze -1) THEN Analyze -1
| |     THEN
| |     Analyze -1)
| |
| 4. a : Assignment
| 5. a' : Assignment
| 6. a' extends a
| 7. a |= x1  ⇒ a' |= x1
| 8. a |≠ x1 ⇒ a' |≠ x1
| 9. a |= x2  ⇒ a' |= x2
| 10. a |≠ x2 ⇒ a' |≠ x2
| ⊢   by Analyze 0 THEN Analyze 0
| |
| |\
| | 11. a |= x1⌈∨⌉x2
| | ⊢  a' |= x1⌈∨⌉x2
| | | by BackThru
| | |    Thm* ∀a:Assignment, q,r:Formula.
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| | |         a |= q⌈∨⌉r  ⟺ a |= q  ∨ a |= r
| | |    {formula_or_sat_lemma}
| | |
| | ⊢  a' |= x1  ∨ a' |= x2
| | | by FwdThru
| | |    Thm* ∀a:Assignment, q,r:Formula.
| | |         a |= q⌈∨⌉r  ⟺ a |= q  ∨ a |= r
| | |    {formula_or_sat_lemma}
| | |    [-1]
| | |    THEN
| | |    Analyze -1
| | |
| | |\
| | | 12. a |= x1
| | | ⊢   by Choose [1] THEN HypBackchain
| | |
| | .\
| | . 12. a |= x2
| | . ⊢   by Choose [2] THEN HypBackchain
| |
| .\
| . 11. a |≠ x1⌈∨⌉x2
| . ⊢  a' |≠ x1⌈∨⌉x2
| . | by FwdThru
| . |    Thm* ∀a:Assignment, q,r:Formula.
| . |         a |≠ q⌈∨⌉r ⟺ a |≠ q & a |≠ r
| . |    {formula_or_falsifiable_lemma}
| . |    [-1]
| . |    THEN
| . |    Sel 2
| . |    (FwdThru
| . |     Thm* ∀a:Assignment, q,r:Formula.
| . |          a |≠ q⌈∨⌉r ⟺ a |≠ q & a |≠ r
| . |     {formula_or_falsifiable_lemma}
| . |     [-2])
| . |
| . 12. a |≠ x1
| . 13. a |≠ x2
| . ⊢
| .   by BackThru
| .      Thm* ∀a:Assignment, q,r:Formula.
| .           a |≠ q⌈∨⌉r ⟺ a |≠ q & a |≠ r
| .      {formula_or_falsifiable_lemma}
| .      THEN
| .      HypBackchain
|
.\
. 2. y1 : Formula
. 3. y2 : Formula
. 4. ∀a,a':Assignment.
. |  a' extends a ⇒ (a |= y1  ⇒ a' |= y1 ) & (a |≠ y1 ⇒ a' |≠ y1)
. 5. ∀a,a':Assignment.
. |  a' extends a ⇒ (a |= y2  ⇒ a' |= y2 ) & (a |≠ y2 ⇒ a' |≠ y2)
. ⊢  ∀a,a':Assignment.
. |  a' extends a ⇒
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. |  (a |= y1⌈⇒⌉y2  ⇒ a' |= y1⌈⇒⌉y2 )
. |  & (a |≠ y1⌈⇒⌉y2 ⇒ a' |≠ y1⌈⇒⌉y2)  by UnivCD
. |
. 6. a : Assignment
. 7. a' : Assignment
. 8. a' extends a
. ⊢  (a |= y1⌈⇒⌉y2  ⇒ a' |= y1⌈⇒⌉y2 )
. |  & (a |≠ y1⌈⇒⌉y2 ⇒ a' |≠ y1⌈⇒⌉y2)
. | by OnHyps [-5;-6]
. |    (λi.
. |     With a (Analyze i) THEN With a' (Analyze -1) THEN Analyze -1
. |     THEN
. |     Analyze -1)
. |
. 4. a : Assignment
. 5. a' : Assignment
. 6. a' extends a
. 7. a |= y1  ⇒ a' |= y1
. 8. a |≠ y1 ⇒ a' |≠ y1
. 9. a |= y2  ⇒ a' |= y2
. 10. a |≠ y2 ⇒ a' |≠ y2
. ⊢   by Analyze 0 THEN Analyze 0
. |
. |\
. | 11. a |= y1⌈⇒⌉y2
. | ⊢  a' |= y1⌈⇒⌉y2
. | | by FwdThru
. | |    Thm* ∀a:Assignment, q,r:Formula.
. | |         a |= q⌈⇒⌉r  ⟺ a |≠ q ∨ a |= r
. | |    {formula_imp_sat_lemma}
. | |    [-1]
. | |    THEN
. | |    BackThru
. | |    Thm* ∀a:Assignment, q,r:Formula.
. | |         a |= q⌈⇒⌉r  ⟺ a |≠ q ∨ a |= r
. | |    {formula_imp_sat_lemma}
. | |    THEN
. | |    Analyze -1
. | |
. | |\
. | | 12. a |≠ y1
. | | ⊢  a' |≠ y1 ∨ a' |= y2   by Choose [1] THEN HypBackchain
. | |
. | .\
. | . 12. a |= y2
. | . ⊢  a' |≠ y1 ∨ a' |= y2   by Choose [2] THEN HypBackchain
. |
. .\
. . 11. a |≠ y1⌈⇒⌉y2
. . ⊢  a' |≠ y1⌈⇒⌉y2
. . | by FwdThru
. . |    Thm* ∀a:Assignment, q,r:Formula.
. . |         a |≠ q⌈⇒⌉r ⟺ a |= q  & a |≠ r
. . |    {formula_imp_falsifiable_lemma}
. . |    [-1]
```

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. . |    THEN
. . |    Sel 2
. . |    (FwdThru
. . |     Thm* ∀a:Assignment, q,r:Formula.
. . |          a |≠ q⌈⇒⌉r ⟺ a |= q  & a |≠ r
. . |     {formula_imp_falsifiable_lemma}
. . |     [-2])
. . |
. . 12. a |= y1
. . 13. a |≠ y2
. . ⊢
. . | by BackThru
. . |    Thm* ∀a:Assignment, q,r:Formula.
. . |         a |≠ q⌈⇒⌉r ⟺ a |= q  & a |≠ r
. . |    {formula_imp_falsifiable_lemma}
. . |
. . ⊢  a' |= y1  & a' |≠ y2  by Analyze 0 THEN HypBackchain
```

END