```
formula_imp_sat_lemma  THM

⊢  ∀a:Assignment, q,r:Formula. a |= q⌜⇒⌝r  ⟺ a |≠ q ∨ a |= r
| by UnivCD
|
1. a : Assignment
2. q : Formula
3. r : Formula
⊢  a |= q⌜⇒⌝r  ⟺ a |≠ q ∨ a |= r   by Analyze 0 THEN Analyze 0
|
|\
| 4. a |= q⌜⇒⌝r
| ⊢  a |≠ q ∨ a |= r   by UnfoldTopAb -1
| |
| 4. (q⌜⇒⌝r under a)  = 3₂
| ⊢   by Rewrite (HigherC valuation_unrollC) -1
| |
| 4. (q under a)  ⇒₃ (r under a)  = 3₂
| ⊢
| | by MoveToConcl -1 THEN GenConcl ((q under a)  = n)
| |    THEN
| |    GenConcl ((r under a)  = m)
| |    THEN
| |    UnivCD
| |
| 4. n : ℕ₃
| 5. (q under a)  = n
| 6. m : ℕ₃
| 7. (r under a)  = m
| 8. n ⇒₃ m = 3₂
| ⊢   by ThreeInd -5 THEN ThreeInd -3 THEN AbReduce -1
| |
| |\
| | 4. (q under a)  = 3₀
| | 5. (r under a)  = 3₀
| | 6. 3₂ = 3₂
| | ⊢   by DNth 1 0
| | |
| | ⊢  a |≠ q  by UnfoldTopAb 0
| |
| |\
| | 4. (q under a)  = 3₀
| | 5. (r under a)  = 3₁
| | 6. 3₂ = 3₂
| | ⊢   by DNth 1 0
| | |
| | ⊢  a |≠ q  by UnfoldTopAb 0
| |
| |\
| | 4. (q under a)  = 3₀
| | 5. (r under a)  = 3₂
| | 6. 3₂ = 3₂
| | ⊢   by DNth 1 0
| | |
| | ⊢  a |≠ q  by UnfoldTopAb 0
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| |\
| | 4. (q under a)  = 3₁
| | 5. (r under a)  = 3₀
| | 6. 3₁ = 3₂
| | ⊢   by ApFunToHypEquands ‘z’ case z: 3₀ → 0; 3₁ → 1; 3₂ → 2; ℤ -1
| | |
| | |\
| | | .....fun wf .....
| | | 7. z : ℕ3
| | | ⊢  case z: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | |    =
| | |    case z: 3₀ → 0; 3₁ → 1; 3₂ → 2;  by ThreeInd -1 THEN AbReduce
```

```
| | |  0
| | |
| | .\
| | . 7. case 3₁: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | .    =
| | .    case 3₂: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | . ⊢   by AbReduce -1
| |
| |\
| | 4. (q under a)  = 3₁
| | 5. (r under a)  = 3₁
| | 6. 3₁ = 3₂
| | ⊢   by ApFunToHypEquands ‘z’ case z: 3₀ → 0; 3₁ → 1; 3₂ → 2; ℤ -1
| | |
| | |\
| | | .....fun wf .....
| | | 7. z : ℕ3
| | | ⊢  case z: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | |    =
| | |    case z: 3₀ → 0; 3₁ → 1; 3₂ → 2;  by ThreeInd -1 THEN AbReduce
```

```
| | |  0
| | |
| | .\
| | . 7. case 3₁: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | .    =
| | .    case 3₂: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | . ⊢   by AbReduce -1
| |
| |\
| | 4. (q under a)  = 3₁
| | 5. (r under a)  = 3₂
| | 6. 3₂ = 3₂
| | ⊢   by DNth 2 0
| | |

| | ⊢  a |= r   by UnfoldTopAb 0
| |
| |\
| | 4. (q under a)  = 3₂
| | 5. (r under a)  = 3₀
| | 6. 3₀ = 3₂
| | ⊢   by ApFunToHypEquands ‘z’ case z: 3₀ → 0; 3₁ → 1; 3₂ → 2; ℤ -1
| | |
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| | |\
| | | .....fun wf .....
| | | 7. z : ℕ3
| | | ⊢  case z: 3_0 → 0; 3_1 → 1; 3_2 → 2;
| | |    =
| | |    case z: 3_0 → 0; 3_1 → 1; 3_2 → 2;  by ThreeInd -1 THEN AbReduce
```

```
| | |  0
| | |
| | .\
| | . 7. case 3₀: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | .    =
| | .    case 3₂: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | . ⊢   by AbReduce -1
| |
| |\
| | 4. (q under a)  = 3₂
| | 5. (r under a)  = 3₁
| | 6. 3₁ = 3₂
| | ⊢   by ApFunToHypEquands ‘z’ case z: 3₀ → 0; 3₁ → 1; 3₂ → 2; ℤ -1
| | |
| | |\
| | | .....fun wf .....
| | | 7. z : ℕ3
| | | ⊢  case z: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | |    =
| | |    case z: 3₀ → 0; 3₁ → 1; 3₂ → 2;  by ThreeInd -1 THEN AbReduce
```

```
| | |  0
| | |
| | .\
| | . 7. case 3₁: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | .    =
| | .    case 3₂: 3₀ → 0; 3₁ → 1; 3₂ → 2;
| | . ⊢   by AbReduce -1
| |
| .\
| . 4. (q under a)  = 3₂
| . 5. (r under a)  = 3₂
| . 6. 3₂ = 3₂
| . ⊢   by DNth 2 0
| . |
| . ⊢  a |= r   by UnfoldTopAb 0
|
.\
. 4. a |≠ q ∨ a |= r
. ⊢  a |= q⌈⇒⌉r   by Analyze -1
. |
. |\
. | 4. a |≠ q
. | ⊢   by OnHyps [-1;0] UnfoldTopAb
. | |
. | 4. (q under a)  = 3₀
. | ⊢  (q⌈⇒⌉r under a)  = 3₂  by AbReduce 0
. | |
. | ⊢  (q under a)  ⇒₃ (r under a)  = 3₂
. | | by HypSubst -1 0 THEN AbReduce 0
```

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. | |
. | ⊢  3₀ ⇒₃ (r under a)  = 3₂  by GenConcl ((r under a)  = n)
. | |
. | 5. n : ℕ₃
. | 6. (r under a)  = n
. | ⊢  3₀ ⇒₃ n = 3₂  by ThreeInd -2 THEN AbReduce 0
. |
. .\
. . 4. a |= r
. . ⊢   by OnHyps [-1;0] UnfoldTopAb
. . |
. . 4. (r under a)  = 3₂
. . ⊢  (q⌈⇒⌉r under a)  = 3₂  by AbReduce 0
. . |
. . ⊢  (q under a)  ⇒₃ (r under a)  = 3₂
. . | by GenConcl ((q under a)  = x)
. . |
. . 5. x : ℕ₃
. . 6. (q under a)  = x
. . ⊢  x ⇒₃ (r under a)  = 3₂  by HypSubst -3 0
. . |
. . ⊢  x ⇒₃ 3₂ = 3₂  by ThreeInd -2 THEN AbReduce 0
END
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