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formula_not_sat_lemma  THM

⊢  ∀F:Formula, a:Assignment. a |= ⌈◇⌉F  ⟺ a |≠ F
| by UnivCD THEN Analyze 0
|
|\
| 1. F : Formula
| 2. a : Assignment
| ⊢  a |= ⌈◇⌉F  ⇒ a |≠ F  by UnivCD
| |
| 3. a |= ⌈◇⌉F
| ⊢  a |≠ F  by Analyze 3 THEN Unfold ‘formula_falsifiable‘ 0
| |
| 3. (⌈◇⌉F under a)  = 3_2
| ⊢  (F under a)  = 3_0  by Reduce -1
| |
| 3. ◇_3(F under a)  = 3_2
| ⊢   by GenConcl ((F under a)  = n)
| |
| 4. n : ℕ_3
| 5. (F under a)  = n
| ⊢  n = 3_0  by MoveToEnd -3 THEN HypSubst -2 -1
| |
| 3. n : ℕ_3
| 4. (F under a)  = n
| 5. ◇_3n = 3_2
| ⊢   by ThreeInd -3
| |
| |\
| | 3. (F under a)  = 3_1
| | 4. ◇_3 3_1 = 3_2
| | ⊢  3_1 = 3_0  by Reduce -1
| | |
| | 4. 3_1 = 3_2
| | ⊢   by ThreeNEQ -1
| |
| .\
| . 3. (F under a)  = 3_2
| . 4. ◇_3 3_2 = 3_2
| . ⊢  3_2 = 3_0  by Reduce -1
| . |
| . 4. 3_0 = 3_2
| . ⊢   by ThreeNEQ -1
|
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. 1. F : Formula
. 2. a : Assignment
. ⊢  a |= ⌈◇⌉F  ⇐ a |≠ F  by Analyze 0
. |
. 3. a |≠ F
. ⊢  a |= ⌈◇⌉F   by OnHyps [0;3] UnfoldTopAb
. |
. 3. (F under a)  = 3_0
. ⊢  (⌈◇⌉F under a)  = 3_2  by Reduce 0
. |
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\begin{array}{l}
\texttt{. } \vdash \ \ \diamond_{\mathbf{3}}\texttt{(F under a)  = 3}_{\mathbf{2}}\texttt{  by HypSubst -1 0 THEN Reduce 0} \\
\texttt{. |} \\
\texttt{. } \vdash \ \ \texttt{3}_{\mathbf{2}}\texttt{ = 3}_{\mathbf{2}}\texttt{  by Auto} \\
\texttt{END}
\end{array}
$$