```
inv_image_ind_tp  THM

⊢  ∀r:(T→T→Prop), S:Type, f:(S→T).
|  WellFnd{i}(T;x,y.r(x,y)) ⇒ WellFnd{i}(S;x,y.r(f(x),f(y)))
| by UnivCD
|
1. T : Type
2. r : T→T→Prop
3. S : Type
4. f : S→T
5. WellFnd{i}(T;x,y.r(x,y))
⊢  WellFnd{i}(S;x,y.r(f(x),f(y)))  by UnfoldTopAb 0
|
⊢  ∀P:(S→Prop).
|  (∀j:S. (∀k:S. r(f(k),f(j)) ⇒ P(k)) ⇒ P(j)) ⇒ (∀n:S. P(n))
| by Unfold `guard` 0 THEN UnivCD
|
6. P : S→Prop
7. ∀j:S. (∀k:S. r(f(k),f(j)) ⇒ P(k)) ⇒ P(j)
8. n : S
⊢  P(n)
| by PushArgs
|    [`f`,(term_to_arg f)
|    ;`T`,(term_to_arg T)
|    ;`n`,(var_to_arg `n`)
|    ;`nn`,(var_to_arg `nn`)
|    ;`S`,(term_to_arg S)
|    ;`i`,(int_to_arg 8)
|    ;`RangeIndTac`,(tactic_to_arg (WFndHypInd 5 -1))]
|
.....main .....
⊢  P(n)
| by Analogy Tactic Directive: %S%
|
|    λp.
|    let x ≡ get_distinct_var `x` p ,
|        n ≡ get_var_arg `n` p ,
|        S ≡ get_term_arg `S` p ,
|        T ≡ get_term_arg `T` p ,
|        f ≡ get_term_arg `f` p in
|    Assert
|    (mk_all_term x T
|     (mk_all_term n S
|      (mk_implies_term
|       (mk_equal_term T (mk_apply_term f (mvt n)) (mvt x))
|       (concl p))))
|    p
|
|\
| .....assertion .....
| ⊢  ∀x:T, n:S. f(n) = x ⇒ P(n)
| | by λp.(Thin (get_int_arg `i` p) THEN Analyze 0) p
| |
| |\
| | .....main .....
```

```
| | 8. x : T
| | ⊢  ∀n:S. f(n) = x ⇒ P(n)  by λp.get_tactic_arg ‘RangeIndTac‘ p p
| | |
| | |\
| | | .....wf .....
| | | 7. ∀j:S. (∀k:S. r(f(k),f(j)) ⇒ P(k)) ⇒ P(j)
| | | ⊢  (λx.∀n:S. f(n) = x ⇒ P(n)) ∈ T→Prop
| | | | by AddHiddenLabel ‘wf‘
| | | |
| | | .....wf .....
| | | ⊢  (λx.∀n:S. f(n) = x ⇒ P(n)) ∈ T→Prop
| | |   by Analogy Tactic Directive: %E%
| | |
| | |      Auto
| | |
| | |\
| | | .....main .....
| | | 8. j : T
| | | 9. ∀k:T. r(k,j) ⇒ (∀n:S. f(n) = k ⇒ P(n))
| | | ⊢  ∀n:S. f(n) = j ⇒ P(n)
| | | | by Analogy Tactic Directive: %L:main%
| | | |
| | | |    (Analyze 0) THEN (Analyze 0)
| | | |
| | | |\
| | | | .....main .....
| | | | 10. n : S
| | | | 11. f(n) = j
| | | | ⊢  P(n)
| | | | | by λp.
| | | | |    let y ≡ get_var_arg ‘n‘ p ,
| | | | |        yy ≡ get_var_arg ‘nn‘ p ,
| | | | |        S ≡ get_term_arg ‘S‘ p ,
| | | | |        r ≡ get_term_arg ‘f‘ p ,
| | | | |        j ≡ var_of_hyp -4 p ,
| | | | |        k,(),T1 ≡ dest_all (h -3 p) ,
| | | | |        k_lt_j,() ≡ dest_implies T1 ,
| | | | |        ryy_lt_ry
| | | | |        ≡ subst
| | | | |          [k,(mk_apply_term r (mvt yy))
| | | | |          ;j,(mk_apply_term r (mvt y))]
| | | | |          k_lt_j ,
| | | | |        P_yy ≡ subst [y,(mvt yy)] (concl p) in
| | | | |    Assert
| | | | |    (mk_all_term yy S (mk_implies_term ryy_lt_ry P_yy))
| | | | |    p
| | | | |

| | | | |\
| | | | | .....assertion .....
| | | | | ⊢  ∀nn:S. r(f(nn),f(n)) ⇒ P(nn)
| | | | | | by (Analyze 0) THEN (Analyze 0)
| | | | | |
| | | | | |\
| | | | | | .....main .....
| | | | | | 12. nn : S
```

```
| | | | | | 13. r(f(nn),f(n))
| | | | | | ⊢  P(nn)
| | | | | | | by λp.
| | | | | | |    let yy ≡ var_of_hyp -2 p ,
| | | | | | |        r ≡ get_term_arg ‘f‘ p ,
| | | | | | |        r_yy ≡ mk_apply_term r (mvt yy) in
| | | | | | |    ((DTerm r_yy -5) THEN (Analyze -1)
| | | | | | |     THEN
| | | | | | |     (DTerm (mvt yy) -1)
| | | | | | |     THEN
| | | | | | |     (Analyze -1)
| | | | | | |     THEN
| | | | | | |     (NthHyp -1))
| | | | | | |    p
| | | | | | |
| | | | | | |\
| | | | | | | .....wf .....
| | | | | | | 9. n : S
| | | | | | | 10. f(n) = j
| | | | | | | 11. nn : S
| | | | | | | 12. r(f(nn),f(n))
| | | | | | | ⊢  f(nn) ∈ T
| | | | | | |   by Analogy Tactic Directive: %E%
| | | | | | |
| | | | | | |      Auto
| | | | | | |
| | | | | | |\
| | | | | | | .....antecedent .....
| | | | | | | 9. n : S
| | | | | | | 10. f(n) = j
| | | | | | | 11. nn : S
| | | | | | | 12. r(f(nn),f(n))
| | | | | | | ⊢  r(f(nn),j)  by RevHypSubst -3 0
| | | | | | | |
| | | | | | | |\
| | | | | | | | .....main .....
| | | | | | | | ⊢  r(f(nn),f(n))  by NthHyp -1
| | | | | | | |
| | | | | | | .\
| | | | | | | . .....wf .....
| | | | | | | . 13. z : T
| | | | | | | . ⊢  r(f(nn),z) ∈ Prop
| | | | | | | .   by Analogy Tactic Directive: %E%
| | | | | | | .
| | | | | | | .      Auto
| | | | | | |
| | | | | | |\
| | | | | | | .....wf .....
| | | | | | | 9. n : S
| | | | | | | 10. f(n) = j
| | | | | | | 11. nn : S
| | | | | | | 12. r(f(nn),f(n))
| | | | | | | ⊢  nn ∈ S  by NthDecl -2
| | | | | | |
| | | | | | .\
```

```
| | | | | | . .....antecedent .....
| | | | | | . 9. n : S
| | | | | | . 10. f(n) = j
| | | | | | . 11. nn : S
| | | | | | . 12. r(f(nn),f(n))
| | | | | | . ⊢  f(nn) = f(nn)
| | | | | | .   by Analogy Tactic Directive: %E%
| | | | | | .
| | | | | | .      Auto
| | | | | |
| | | | | |\
| | | | | | .....wf .....
| | | | | | 12. nn : S
| | | | | | ⊢  r(f(nn),f(n)) ∈ Prop
| | | | | |   by Analogy Tactic Directive: %E%
| | | | | |
| | | | | |      Auto
| | | | | |
| | | | | .\
| | | | | . .....wf .....
| | | | | . ⊢  S ∈ Type
| | | | | .   by Analogy Tactic Directive: %E%
| | | | | .
| | | | | .      Auto
| | | | |
| | | | .\
| | | | . .....main .....
| | | | . 12. ∀nn:S. r(f(nn),f(n)) ⇒ P(nn)
| | | | . ⊢  P(n)  by OnHyps [-2;-3;-3] Thin
| | | | . |
| | | | . .....main .....
| | | | . 8. n : S
| | | | . 9. ∀nn:S. r(f(nn),f(n)) ⇒ P(nn)
| | | | . ⊢  P(n)
| | | | .   by Analogy Tactic Directive: %E%
| | | | .
| | | | .      BackThru 7

| | | |
| | | |\
| | | | .....wf .....
| | | | 10. n : S
| | | | ⊢  (f(n) = j) ∈ Prop
| | | |   by Analogy Tactic Directive: %E%
| | | |
| | | |      Auto
| | | |
| | | .\
| | | . .....wf .....
| | | . ⊢  S ∈ Type
| | | .   by Analogy Tactic Directive: %E%
| | | .
| | | .      Auto
| | |
| | |\
| | | .....wf .....
```

```
| | | 8. j : T
| | | ⊢  (∀k:T. r(k,j) ⇒ (∀n:S. f(n) = k ⇒ P(n))) ∈ Prop
| | | | by AddHiddenLabel ‘wf‘
| | | |
| | | .....wf .....
| | | ⊢  (∀k:T. r(k,j) ⇒ (∀n:S. f(n) = k ⇒ P(n))) ∈ Prop
| | |   by Analogy Tactic Directive: %E%
| | |
| | |      Auto
| | |
| | .\
| | . .....wf .....
| | . 7. ∀j:S. (∀k:S. r(f(k),f(j)) ⇒ P(k)) ⇒ P(j)
| | . ⊢  T ∈ Type
| | .   by Analogy Tactic Directive: %E%
| | .
| | .      Auto
| |
| .\
| . .....wf .....
| . 7. ∀j:S. (∀k:S. r(f(k),f(j)) ⇒ P(k)) ⇒ P(j)
| . ⊢  T ∈ Type
| .   by Analogy Tactic Directive: %E%
| .
| .      Auto
|
.\
. .....main .....
. 9. ∀x:T, n:S. f(n) = x ⇒ P(n)
. ⊢  P(n)
. | by λp.
. |    let y ≡ get_var_arg ‘n‘ p ,
. |        r ≡ get_term_arg ‘f‘ p in
. |    ((DTerm (mk_apply_term r (mvt y)) -1)
. |     THEN
. |     (DTerm (mvt y) -1 THENA Declaration)
. |     THEN
. |     (Analyze -1)
. |     THEN
. |     Hypothesis)
. |    p
. |
. |\
. | .....wf .....
. | 8. n : S
. | ⊢  f(n) ∈ T
. |   by Analogy Tactic Directive: %E%
. |
. |      Auto
. |
. .\
. . .....antecedent .....
. . 8. n : S
. . ⊢  f(n) = f(n)
. .   by Analogy Tactic Directive: %E%
```

```
. .
. .       Auto
END
```