```
wellfounded_functionality_wrt_iff  THM

⊢  ∀r1:(T1→T1→Prop), r2:(T2→T2→Prop).
|  T1 = T2
|  ⇒
|  (∀x,y:T1. r1(x,y) ⟺ r2(x,y))
|  ⇒
|  (WellFnd{i}(T1;x,y.r1(x,y)) ⟺ WellFnd{i}(T2;x,y.r2(x,y)))
| by UnivCD
|
1. T1 : Type
2. T2 : Type
3. r1 : T1→T1→Prop
4. r2 : T2→T2→Prop
5. T1 = T2
6. ∀x,y:T1. r1(x,y) ⟺ r2(x,y)
⊢  WellFnd{i}(T1;x,y.r1(x,y)) ⟺ WellFnd{i}(T2;x,y.r2(x,y))
| by Analyze 0
|
|\
| ⊢  WellFnd{i}(T1;x,y.r1(x,y)) ⇒ WellFnd{i}(T2;x,y.r2(x,y))
| | by BackThru
| |    Thm* ∀r1:(T1→T1→Prop), r2:(T2→T2→Prop).
| |         T1 = T2
| |         ⇒
| |         (∀x,y:T1. r1(x,y) ⇐ r2(x,y))
| |         ⇒
| |         WellFnd{i}(T1;x,y.r1(x,y)) ⇒ WellFnd{i}(T2;x,y.r2(x,y))
| |    {wellfounded_functionality_wrt_implies}
| |
| |\
| | ⊢  T1 = T2  by Trivial
| |
| .\
| . ⊢  ∀x,y:T1. r1(x,y) ⇐ r2(x,y)
| .   by (Unfold ‘guard‘ 0) THEN UnivCD THEN (BackThru 6)
|
.\
. ⊢  WellFnd{i}(T1;x,y.r1(x,y)) ⇐ WellFnd{i}(T2;x,y.r2(x,y))
. | by BackThru
. |    Thm* ∀r1:(T1→T1→Prop), r2:(T2→T2→Prop).
. |         T1 = T2
. |         ⇒
. |         (∀x,y:T1. r1(x,y) ⇐ r2(x,y))
. |         ⇒
. |         WellFnd{i}(T1;x,y.r1(x,y)) ⇒ WellFnd{i}(T2;x,y.r2(x,y))
. |    {wellfounded_functionality_wrt_implies}
. |
. |\
. | ⊢  T2 = T1  by Trivial
. |
. .\
. . ⊢  ∀x,y:T2. r2(x,y) ⇐ r1(x,y)
. .   by (Unfold ‘guard‘ 0) THEN UnivCD THEN (BackThru 6)
END
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