```
wellfounded_functionality_wrt_implies  THM

⊢  ∀r1:(T1→T1→Prop), r2:(T2→T2→Prop).
|  T1 = T2
|  ⇒
|  (∀x,y:T1. r1(x,y) ⇐ r2(x,y))
|  ⇒
|  WellFnd{i}(T1;x,y.r1(x,y)) ⇒ WellFnd{i}(T2;x,y.r2(x,y))
| by Unfolds [`wellfounded`;`guard`] 0 THEN UnivCD
|
1. T1 : Type
2. T2 : Type
3. r1 : T1→T1→Prop
4. r2 : T2→T2→Prop
5. T1 = T2
6. ∀x,y:T1. r1(x,y) ⇐ r2(x,y)
7. ∀P:(T1→Prop).
|  (∀j:T1. (∀k:T1. r1(k,j) ⇒ P(k)) ⇒ P(j)) ⇒ (∀n:T1. P(n))
8. P : T2→Prop
9. ∀j:T2. (∀k:T2. r2(k,j) ⇒ P(k)) ⇒ P(j)
⊢  ∀n:T2. P(n)  by SubstClause (∀n:T1. P(n)) 0
|
|\
| .....equality .....
| ⊢  (∀n:T2. P(n)) = (∀n:T1. P(n))  by Repeat Analyze
|
.\
. ⊢  ∀n:T1. P(n)  by BackThru 7 THEN Thin 7
. |
. 7. P : T2→Prop
. 8. ∀j:T2. (∀k:T2. r2(k,j) ⇒ P(k)) ⇒ P(j)
. 9. j : T1
. 10. ∀k:T1. r1(k,j) ⇒ P(k)
. ⊢  P(j)  by BackThru 8 THEN Thin 8
. |
. 8. j : T1
. 9. ∀k:T1. r1(k,j) ⇒ P(k)
. 10. k : T2
. 11. r2(k,j)
. ⊢  P(k)  by BackThru 9 THEN Thin 9
. |
. 9. k : T2
. 10. r2(k,j)
. ⊢  r1(k,j)  by BackThru 6
END
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